PREFACE

The Analytic Hierarchy Process, McGraw Hill International, New York, 1980.

The decision-maker, be he motivated by the need to predict or to control, usually faces a complex system of interrelated components, such as resources, desired outcomes or objectives, persons or groups of persons, etc.: he is interested in analyzing this system. Presumably, the better he understands this complexity, the better his prediction or decision will be. In this book we present a theory, whose application reduces the study of even formidably intricate systems to a sequence of pairwise comparisons of properly identified components.

This theory had its beginnings in the fall of 1971 while I was working on problems of contingency planning for the Department of Defense. It had its adolescence in 1972 in a study for the NSF (and later also ERDA) on rationing electricity to industries according to their contribution to the welfare of the nation. The origins of the scale which relates judgments to numbers date back to the serious events of June and July, 1972, in Cairo while I was there analyzing the effect of "No Peace, No War" on Egypt's economic, political, and military status.

The application maturity of the theory came with the Sudan Transport Study in 1973, which I was directing. Its theoretical enrichment was happening all along the way, with greatest intensity between 1974 and 1978. The applications so far have been many and varied, ranging from an analysis of terrorism for the Arms Control and Disarmament Agency (published in a book edited by Dr. Robert Kupperman of the Agency) where I worked in Washington for seven years, and several other studies of conflict (e.g., the conflict in Northern Island) to allocating resources according to priority for large private, governmental, and international concerns.

The theory reflects what appears to be an innate method of operation of the human mind. When presented with a multitude of elements, controllable or not, which comprise a complex situation, it aggregates them into groups, according to whether they share certain properties. Our model of this brain function allows a repetition of this process, in that we consider these groups, or rather their identifying common properties, as the elements of a new level in the system. These elements may, in turn, be grouped according to another set of properties, generating the elements of yet another "higher" level, until we reach a single "top" element which can often be identified as the goal of our decision-making process.

What we have just described is commonly called a hierarchy, i.e., a system of stratified levels, each consisting of so many elements, or factors. The central question is, in terms of this hierarchy: how strongly do the individual factors of the lowest level of the hierarchy influence its top factor, the overall goal? Since this influence will not be uniform over the factors, we are led to the identification of their intensity or, as we prefer to call it, their priorities.

This determination of the priorities of the lowest factors relative to the goal, can be reduced to a sequence of priority problems, one for each level, and each such priority problem to a sequence of pairwise comparisons. These comparisons remain the central ingredients to our theory, even if the original problem should have been complicated by feedback relations between various levels or factors.

Let us return to our suggestion that our theory is a model of the way in which the human mind conceptualizes and structures a complicated problem. We were influences by the following observations:

(1) When we watch people participating in the process of structuring and prioritizing a hierarchy, we find that they engage naturally in successive grouping of items within levels and in distinguishing among levels of complexity.

(2) Individuals informed about a particular problem may structure it hierarchically somewhat differently, but if their judgments are similar, their overall answers tend to be similar. Also, the process is robust. In other words, fine distinctions within the hierarchy tend in practice not to be decisive.

(3) In the course of developing the theory we find a mathematically reasonable way to handle judgments.

Participants tended to find that the process captures their intuitive understanding of a problem. Furthermore, the psychological limits seem to be consonant with conditions for mathematical stability of the results.

In his beautiful book *Number the Language of Science*, The Macmillan Company, New York (3rd edition), Dantzig observes that the human mind has a sense for numbers which is primitive and predates true counting; namely, the ability to recognize that a small collection of objects has increased or decreased when things are added to it or subtracted from it. This is an intuitive talent which is not the same as counting. He points out that individuals and even some animals have this talent. Finally, he speculates on whether the concept is born of experience or whether experience merely serves to render explicit what is already latent in mind. On reflection it appears that it is the latter; that consciousness is a process of identifying events and distinguishing the intensity or degree of differences among them according to whatever properties they have in common. Thus, what we know as "qualitative" is a fuzzy way of acknowledging differences. Since our survival requires that we be more specific, we have developed the talent of number sense.

When a single experience involves a variety of different sensations or activities and some kind of integrated interpretation or action is needed, these activities must be combined somehow. How we combine them depends on the purpose they are supposed to serve; our objectives dictate where we place the emphasis. We need the idea of priority and its measurement.

The methodology should then be useful to model problems incorporating knowledge and judgments in such a way that the issues involved are clearly articulated, evaluated, debated, and prioritized. The judgments can be refined through a continuous application of a feed back process, each application leading to a refinement and sharpening of the judgments. We have even used the Analytic Hierarchy Process to obtain group judgments through consensus. There is no such thing as *the* answer but *an* answer, which with constant exposure, develops into *the* answer for the decision-maker. In whatever form the final judgment is cast, there will always be people whose judgments differ from any particular outcome but when a group was involved in formulating judgments a synthesis of interests would have been created.

We show that age of the old adage that one cannot compare apples and oranges is not true. An apple and an orange have many characteristics in common: size, shape, taste, aroma, color, seediness, juiciness, and so on. We may prefer an orange for some characteristics and an apple for others. In addition, the strength of our preference for these characteristics may vary. We may indifferent to size and color but may have a strong preference for taste which again may vary with the time of day. It is our thesis that this sort of complicated comparison occurs in real life over and over again, and some kind of a mathematical approach is required. We will also develop a dynamic method for such comparisons.

The practice of decision-making is concerned with weighting alternatives, all of which fulfill a set of desired objectives. The problem is to choose that alternative which most strongly fulfills the entire set of objectives. We are interested in deriving numerical weights for alternatives with respect to sub-objectives and for sub-objectives with respect to higher order objectives. We would like these weights to be meaningful for allocating resources. For example, if they are derived to represent the value of money or distance or whatever physical quantity is being considered, they should be the same, or close to, what an economist or physicist may obtain using his methods of measurement. Thus our process of weighting should produce weights or priorities that are estimated of an understanding ratio scale. At the same time in situations with physical interdependence among activities, high priority activities which depend on low priority ones must not inadvertently be short-changed by reducing the resource allocation to the low priority ones. That is why resource allocation must be made subject to interdependence constraints.

Even with the same constraints there exist a variety of decision-making styles. One Korean economic planner –the man who thought his country should do better than Japan- said:

> In Japan, the decision process is talk, talk, talk, until you reach consensus. In Korea and in China, it is talk, talk, but then somebody on top makes a decision. You see in the humblest Korean peasant's home, where he is master. You see it, and criticize, in our politics. We see in our big business where there is excellent research, but the final decision is the president's. This can create problems as our pattern of industry becomes more complicated. But it is very good in the early states of industrial growth.

This fits well with the comments of a senior Japanese civil servant on the decision process in Japan:

> Every decision in Japan is taken by consensus. In Japanese government, most policies are originally suggested by officials. Their suggestions then go through many forums, of which the cabinet is only the last. Even in cabinet, talk can continue for hours without anybody being very precise. Then at the end, the prime minister says: this is our consensus. He is not very precise either. But action can then be taken in line with that unprecise consensus, and with everybody feeling he has had some say in what being done. The consequence is that in Japan every decision is mediocre. But execution is then excellent. (*The Economist*, 7 May 1977, p. 46.)

Sometimes decisions taken by large organizations or governments appear to ignore human beings. Emilio Daddario in one of his papers says that:

Because of the uncertainties of political decision-making, society does not perform up to its technical capabilities ….. Until the political process provides a clear ranking of priorities, the contributions of science and technology to specific problems of public welfare will probably remain random and unsystematic … The political decision-makers must take care that, in adopting a systematic approach to the ranking of priorities, they do not abdicate their primary function of defending human values. In using this approach to solve large social problems, the decision-makers are learning how to resolve quantitatively many decisions previously left to intuitive or normative judgments. (Emilio Dadario, *Ventures*, Magazine of the Yale Graduate School, Spring, 1971)

Perhaps, our quantitative approach is a process which avoids the dehumanizing dangers of which Daddario speaks.

This book is intended for readers of diverse backgrounds and intentionally involves some repetition of ideas. It is not solely aim at people doing research or colleagues who have distinguished themselves in the area of measurement.

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PREFACE TO THE SECOND REVISED EDITION

The reader is to consult the new materials of this second extended edition in the nearly 200-page addendum, particularly those relating to the axioms, and to absolute measurement. The question of rank behavior is covered in some detail. It is interesting and reveals how relative measurement, which may permit rank reversal, compares with absolute measurement, which legislates rank preservation by the nature of its normative standards. As of this writing, there have appeared two bibliographies on the subject. There have been three special issues of journals: Socio-Economic Planning Sciences, December, 1986, Mathematical Modeling, August, 1987, and the European Journal of Operations Research, August, 1990. Some of the other books on the topic which have appeared both in English and in other languages are listed inside the back cover.

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