INTRODUCTION TO THE ANP AND ITS SUPERMATRICES

In this introduction we review the ANP process and the *SuperDecisions* software and show some applications. Applications may be simple, consisting of a single network, or complex, consisting of a main network and two or more layers of sub-networks. Each network and sub-network is created in its own window.
In practice an application consists of:

- **A Simple Network –** *HamburgerModel-* All the clusters and their nodes are in a single window. An example of a simple network would be a "market share" application such as the Hamburger model. The simple network itself is the decision network because it contains the cluster of nodes that serve as the alternatives of the decision. In a market share application they are the competitors for whom market share is being predicted; for example, McDonald's, Burger King and Wendy's.
- **A Two-level Network -** *Car Purchase BCR* -There is a top-level network with Merit nodes such as Benefits, Opportunities, Costs and Risks, each of which has a sub-network. The alternatives cluster is in each of the sub-networks. The sub-networks are the decision networks because they contain the alternatives.
- **A Complex Network** - National Missile Defense Model -There is a main network of Merits nodes (Benefits, Opportunities, Costs and Risks), each having an attached sub-network that contains among others nodes that will serve as control criteria. The nodes selected to serve as control criteria, the high priority nodes in the network, have decision networks containing the alternatives attached to them. In practice this is the most complex system we work with, though there is no theoretical limitation on the number of levels of sub-networks.

A model is contained in a network system that physically is a single file. All the networks and sub-networks are in the same file. The file has the extension *.sdmod*; for example, the *Hamburger* model is in the file named *Hamburger.sdmod*. If you have the *SuperDecisions* software installed a model file can be launched by double-clicking on it within Windows Explorer. Sometimes the files for the Encyclicon applications have been saved in an old *.mod* format or a zipped format with a *.mod.gz* extension to reduce their sizeSuch a zipped file cannot be launched by double-clicking. Use the File Open command in *SuperDecisions* to open the file.

There are four sample models that we will discuss in the introduction: *Bridge*, *Hamburger*, *Car Purchase BCR,* and *National Missile Defense*. All can be found in the software under the SuperDecisions Help>Sample Models>Introductory Models command. The *Bridge* example is a simple network used to illustrate the idea of feedback. The *Hamburger* example is a simple network used to estimate market share of fast food restaurants. The *Car Purchase BCR* model is a two-level network with Benefits, Costs and Risks nodes in the top-level network, and the alternatives in the sub-networks attached to them. They range from the simplest example of feedback to a complex multi-level network structure. All of these examples are included in the sample models of the *SuperDecisions* software. To load a sample model use the Help, Sample Models command in the software and select the one you want. Sample models are located in the c:/program files/super decisions/samples directory.

Bridge is the simplest example in a single network and we use it to demonstrate the supermatrix idea.

Hamburger is used to estimate the market share of three fast-food hamburger places. The model consists of a single network containing the factors that consumers consider when choosing a fast-food restaurant. Some of its clusters are inner dependent with the nodes in them being compared with respect to other nodes in the same cluster. It has cluster comparisons as well as node comparisons. We use this model to explain more complicated supermatrices, inner and outer dependence and the motivation behind doing cluster comparisons. It is a simple network model in a single window.

Car Purchase BCR is a model for selecting the best kind of car to buy: Japanese, European or American, by taking into consideration the benefits, costs and risks of each type of car. It is a complex two-layer model with three sub-networks. The top-level network contains a benefits node, a costs node and a risks node, each of which has a sub-network where the alternatives are located. Judgments in a sub-network are made from the perspective of its controlling node in the network above.

National Missile Defense is the most complex kind of application with a top-level control model in which the priorities of the BOCR have been obtained by rating them against the US's national objectives. It has control criteria sub-networks under that and finally decision networks containing the alternatives at the bottom.

DEMONSTRATION OF THE SIMPLEST TYPE OF FEEDBACK NETWORK, THE BRIDGE MODEL

Bridge is a decision problem to pick the best of two bridge designs. It is a simple network of one level that contains only two clusters, with two nodes in each cluster, and links between the nodes. A network is structured of clusters, nodes and links. We use this model to show how feedback arises in a network decision structure and how the pairwise comparison questions are formulated when there is feedback. Here the clusters are outer dependent, that is, nodes in a cluster are compared only with respect to nodes in the other cluster.

Load the *Bridge* model by selecting Help, Sample Models from the main menu and selecting *bridge.mod*

The *Bridge* model, a simple network model, is shown in Figure 1. Clusters may be re-sized by left-clicking on the small button at the lower right hand corner and dragging. To select a cluster left-click on the title bar. A cluster is selected when its title bar is highlighted. Left-click on the title bar of a cluster window and drag to move it to a different location.

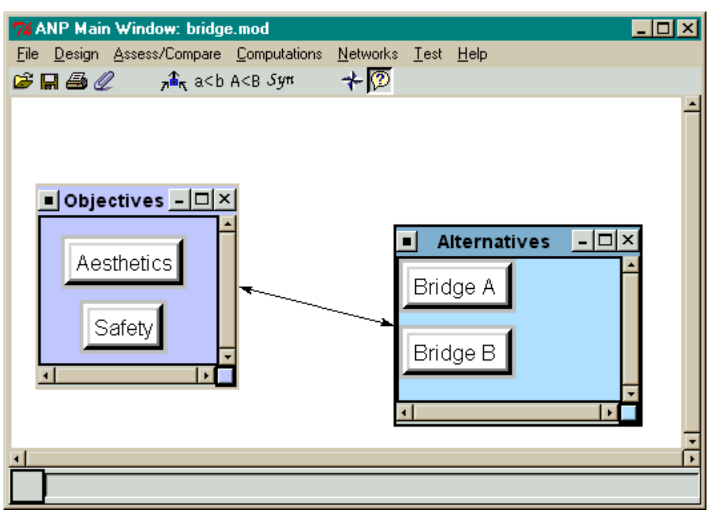


Figure 1. The *Bridge* Model: a Simple Network.

The decision in this model is to select the best bridge. The objectives are to have a safe bridge and an aesthetically pleasing bridge. If one were doing the model from the top down as in a hierarchy, there would be a goal with Aesthetics and Safety as the criteria and Bridge A and Bridge B as the alternatives. One would compare Aesthetics to Safety, Safety would likely be perceived as extremely more important, so the safest bridge would be the "best" choice.

But in a feedback network one compares the bridges for preference with respect to Aesthetics and to Safety, and one also compares the prevalence of Aesthetics versus Safety for each bridge. The net result of this is that priorities are obtained for all four nodes in the system. Suppose the safest bridge, B, is unattractive, and the nicer looking bridge, A, is very safe, though not as safe as B. The priorities of the criteria depend on the bridges available and since both are quite safe, the priority of safety in the feedback system ends up less than it would be in a hierarchy where one compares Safety to Aesthetics in the abstract and apart from any specific bridge. It makes common sense that if both bridges are very safe, one should pick the better-looking bridge, even though one bridge is far safer than the other. The Analytic Network Process through feedback guides us to the best choice in a way that matches our common sense.

The Aesthetics node in the Objectives cluster is linked to Bridge A and to Bridge B, and because there is a link between nodes, a link appears from the Objectives cluster to the Alternatives cluster. Because at least one node in the Objectives cluster is linked to nodes in the Alternatives cluster, a link appears automatically from the Objectives cluster to the Alternatives cluster. Aesthetics is the parent node and Bridge A will be compared to Bridge B with respect to it. The node Safety is also linked to Bridge A and Bridge B, and they will be compared for preference with respect to safety.

To turn on the “show connections” mode, as shown in Figure 2 click on the star-shaped icon . When you place your cursor over a node when this icon is depressed, the nodes connected from it will be outlined in red. Try this by placing the cursor over the Bridge A node and the Aesthetics and Safety nodes will be outlined in red.

When a node has had the comparisons marked as completed for nodes within a cluster that are connected to it, the cluster window of these nodes will also be outlined in red. Both bridges are also connected to the nodes in the Objectives cluster, so holding the cursor over the Bridge A node will show Aesthetics and Safety outlined in red, and the Objectives cluster being outlined in red indicates that the comparison of these nodes with respect to Bridge A is complete.

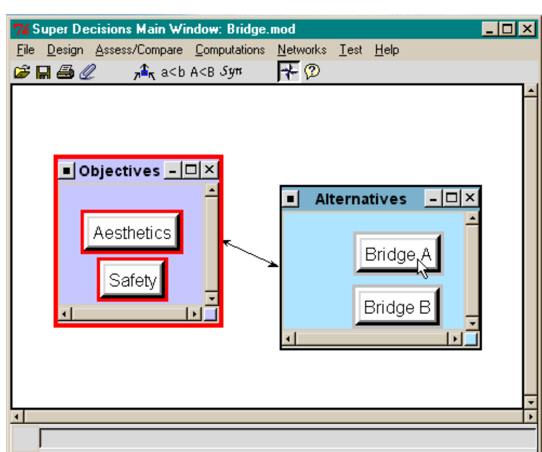


Figure 2. The Aesthetics Node is connected to Bridge A and Bridge B.

FEEDBACK LINKS

It is easy for those who have used the Analytic Hierarchy Process to understand how to pairwise compare Bridge A and Bridge B with respect to Aesthetics. Bridge A would be highly preferred. But what may be new is the idea that criteria may be compared with respect to an alternative. What does that mean? When comparing, for example, Aesthetics and Safety with respect to Bridge A, the question is: What is more a more pronounced or prevalent characteristic of Bridge A, its aesthetics or its safety? Bridge A is beautiful and that is what we like best about it. Its safety, though quite adequate, is nothing notable. So we strongly prefer its aesthetics to its safety.

For Bridge B what is its more preferable characteristic, aesthetics or safety? Since it is quite ugly, the answer is that the Safety of Bridge B is extremely preferable to its Aesthetics. These kinds of preference questions and answers, both directions, help us establish our true priorities for *all* the elements in the problem.

THE SUPERMATRIX

The comparison process will be covered in the next demonstration. Here we will show the various computations involving the supermatrix. To show the three different supermatrices, select the Computations command from the menu shown in Figure 3.

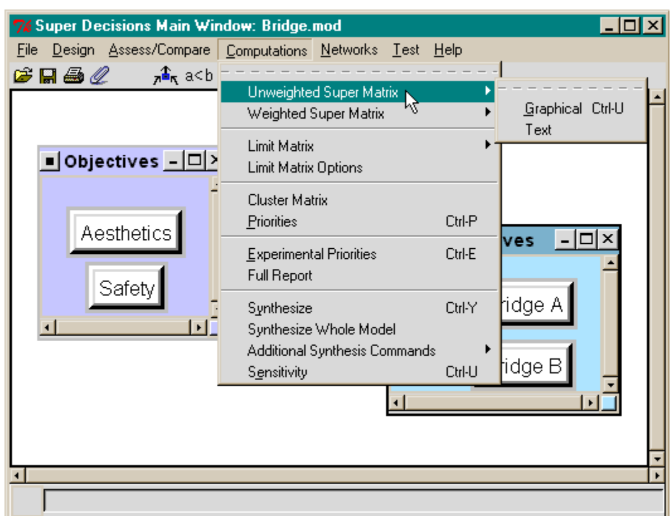


Figure 3. The Computations Menu.

THE UN-WEIGHTED, WEIGHTED AND LIMIT SUPERMATRICES

There are three supermatrices associated with each network: the Unweighted Supermatrix, the Weighted Supermatrix and the Limit Supermatrix. Supermatrices are arranged with the clusters in alphabetical order across the top and down the left side, and with the elements within each cluster in alphabetical order across the top and down the left side. To change the ordering in a supermatrix, you need only re-name the clusters and/or the elements, so their alphabetical order will be the order you want. Changing names after building a model and making comparisons is permitted and will correctly preserve any judgments that have been made.

The unweighted supermatrix is composed of column vectors that are the priorities obtained by comparing nodes is a cluster with respect to a parent node. The column for a given node contains all the priority vectors in the system with that node as a parent of the comparison. A parent node may have children in many different clusters, so the priority vectors are stacked on top of each other in the parent node's column. Each priority vector sums to 1.0, so the numbers in the column of a given node may sum to more than 1, though the sum will always be an integer (or zero if that node is not connected to any other nodes in the entire system)..

Each column has to sum to 1 for a supermatrix to converge to the limit supermatrix. This is done by weighting all the numbers in each "component" of the supermatrix by the cluster priorities. If the clusters have not been pairwise compared, their priorities are assumed to be equal. The [A,B] component of a supermatrix, for example, is the submatrix consisting of the priority vectors from pairwise comparing elements in the B cluster linked from parents in the A cluster. sets with the parent in the A cluster and the children in the B cluster.

When you multiply an unweighted supermatrix component by a constant, you multiply all the numbers in the component by the same constant.

The effect of doing this is that all the columns become stochastic, that is, they sum to 1.0, and the supermatrix will now converge.

The unweighted supermatrix contains the local priorities derived from the pairwise comparisons throughout the network as shown in Figure 4. For example, the priorities of the elements Aesthetics and Safety, with respect to Bridge A are shown in the two bottom cells of the first column, 0.857143 and 0.142857. This may be interpreted with the statement, "The Aesthetics characteristic of Bridge A is between strongly and very strongly, or 6 times, more its dominant preferred characteristic than its Safety aspect." All the local priority information can be read directly from the unweighted Supermatrix.

ANP Main Window: bridge.mod: Unweighted Super ...

Cluster Node Labels		Alternatives		Objectives	
		Bridge A	Bridge B	Aesthetics	Safety
Alternatives	Bridge A	0.000000	0.000000	0.875000	0.333333
	Bridge B	0.000000	0.000000	0.125000	0.666667
Objectives	Aesthetics	0.857143	0.100000	0.000000	0.000000
	Safety	0.142857	0.900000	0.000000	0.000000

Done

Figure 4. The Unweighted Supermatrix for the Bridge Model.

Definition of Component: A component in a supermatrix is the block defined by a cluster name at the left and a cluster name at the top. For example, the (Alternatives, Alternatives) component in Figure 4 is composed of the block of four zeros in the upper left-hand corner shown in the screen clip below.

Detail of (Alternatives, Alternatives) Component from Figure 4

Cluster Node Labels		Alternatives	
		Bridge A	Bridge B
Alternatives	Bridge A	0.000000	0.000000
	Bridge B	0.000000	0.000000

The (Alternatives, Objectives) component is the block of four numbers in the left hand bottom corner of Figure 4 as shown in the screen clip below:

Detail of (Objectives, Alternatives) Component from Figure 4

Objectives	Aesthetics	0.857143	0.100000
	Safety	0.142857	0.900000

The weighted supermatrix is obtained by multiplying all the elements in a component of the unweighted supermatrix by the corresponding cluster weight. We will say more about cluster weights when we demonstrate the *Hamburger* model later. In this example there were no cluster comparisons, since there are only two clusters and cluster comparisons cannot be made when there are only two. The weighted and unweighted supermatrices are the same in this example and are shown in Figure 5. Notice that as the columns already sum to one in the unweighted supermatrix there is no need to weight the components to make the columns sum to one in the weighted supermatrix.

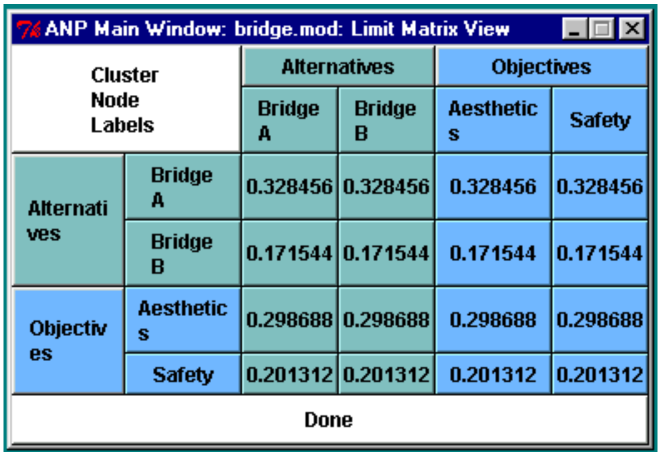
ANP Main Window: bridge.mod: Weighted Super Ma...

Cluster Node Labels		Alternatives		Objectives	
		Bridge A	Bridge B	Aesthetics	Safety
Alternatives	Bridge A	0.000000	0.000000	0.875000	0.333333
	Bridge B	0.000000	0.000000	0.125000	0.666667
Objectives	Aesthetics	0.857143	0.100000	0.000000	0.000000
	Safety	0.142857	0.900000	0.000000	0.000000

Done

Figure 5. The Weighted (same as Unweighted) Supermatrix for the Bridge Model.

The limit supermatrix is obtained by raising the weighted supermatrix to powers by multiplying it times itself. When the column of numbers is the same for every column, the limit matrix has been reached and the matrix multiplication process is halted. The limit supermatrix for the *Bridge* Model is shown in Figure 6.



ANP Main Window: bridge.mod: Limit Matrix View

Cluster Node Labels		Alternatives: Bridge A	Alternatives: Bridge B	Objectives: Aesthetics	Objectives: Safety
Alternatives	Bridge A	0.328456	0.328456	0.328456	0.328456
	Bridge B	0.171544	0.171544	0.171544	0.171544
Objectives	Aesthetics	0.298688	0.298688	0.298688	0.298688
	Safety	0.201312	0.201312	0.201312	0.201312

Done

Figure 6. The Limit Supermatrix for the Bridge Model.

The Computations Priorities command on the menu displays the priorities in two ways: as they appear in the supermatrix, and with the priorities normalized by cluster as shown in Figure 7. The columns of the limit supermatrix are all the same, so the priorities for all the nodes can be read from any column.

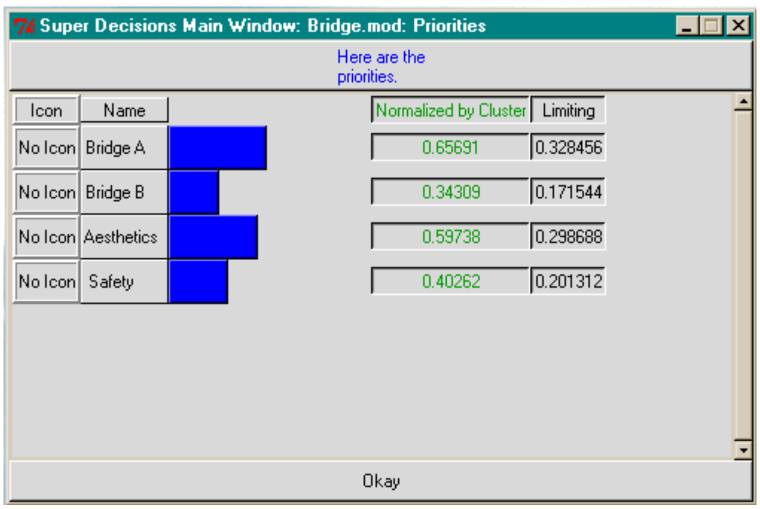


Figure 7. The Priorities from the Limit Supermatrix

The Computations Synthesize command displays the final results in three ways as shown in Figure 8. The *Raw* column gives the priorities from the limiting supermatrix (which also appear in the *Limiting* column above), the *Normals* column shows the results normalized for each component (which also appear in the *Normalized by Cluster* column above) and the *Ideals* column shows the results obtained by dividing the values in either the normalized or limiting columns by the largest value in the column.

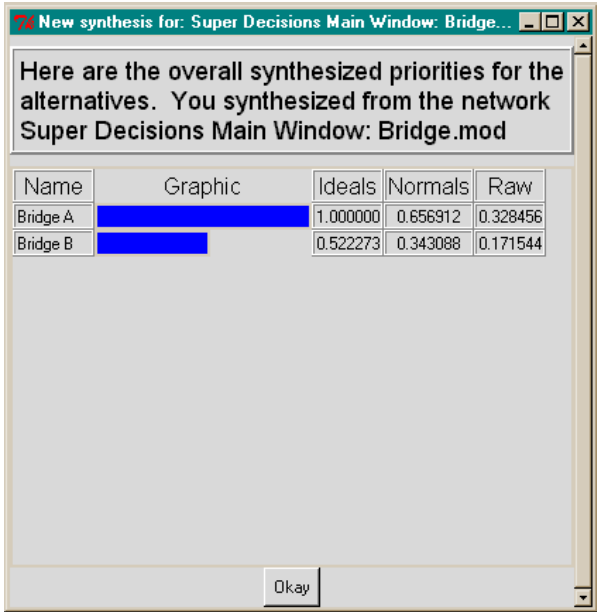


Figure 8. The Synthesized Values – the Results for the Alternatives.

The results show that Bridge A is best. This matches our intuition: "Bridge A is very safe and also the best looking, so choose it. Bridge B has overkill so far as safety is concerned, and it is not good looking – so it is not the best bridge overall. Do not choose it."

DEMO OF THE HAMBURGER MODEL - A SIMPLE NETWORK MODEL

The purpose of the *Hamburger* model, a simple network application shown in Figure 9, is to estimate the market share of three fast-food hamburger joints. A simple network has all the clusters and their nodes in a single window. There are no sub-networks. All the comparison questions are asked from the perspective of what is more important or preferred with respect to market share. In this demo we will explain the motivation behind doing cluster comparisons.

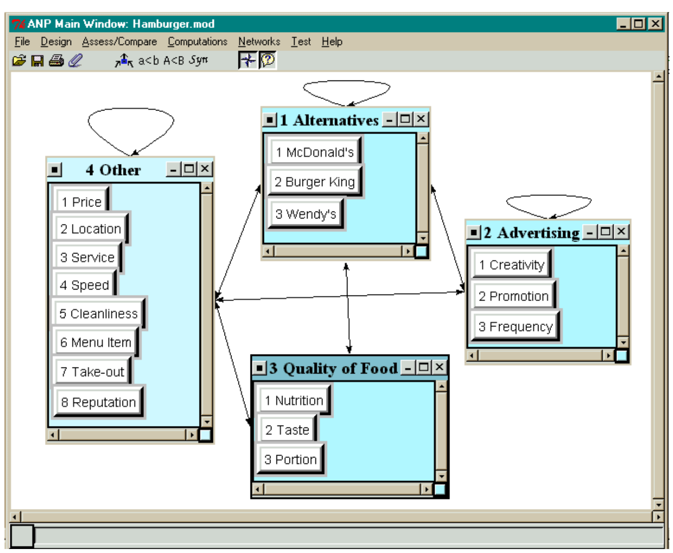


Figure 9. The *Hamburger* Model for Predicting Market Share.

In this model the loops indicate inner dependence among the elements in the cluster. In Figure 10 is a view of the model with icons instead of cluster windows. To switch back and forth from a cluster icon to a cluster window double left click with your mouse on it.

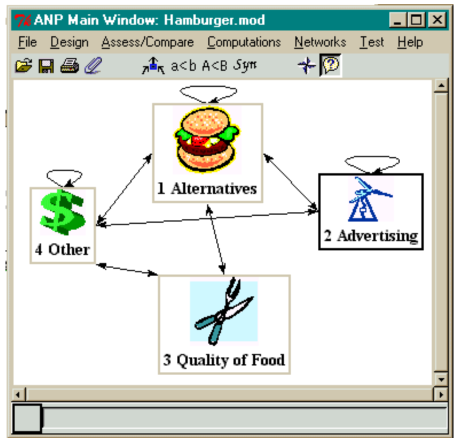


Figure 10. An Iconized View of the Hamburger Model.

Shortcut to Cluster menu: Right click on the background within any cluster to get a drop-down menu of cluster commands as shown in Figure 11 below.

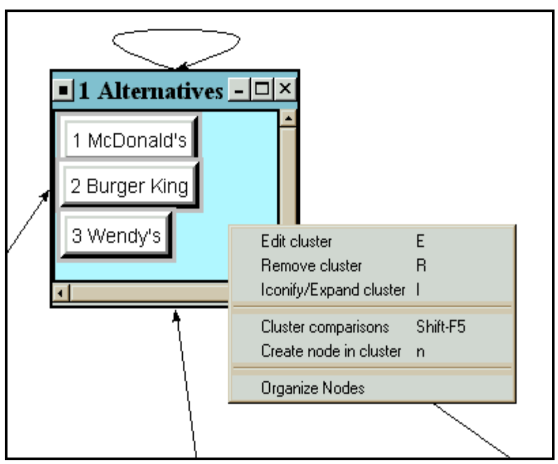


Figure 11. Drop-down Menu of Cluster Commands.

Select the Edit cluster command to change fonts, choose colors or icons, or type definitions. The Organize Nodes command can be used to find missing nodes that may have accidentally scrolled off the window.

COMPARISONS

Pairwise comparisons for the nodes in each cluster that belong to a parent node are carried out for all the parent nodes in the model. Select the Assess/Compare command, then select the cluster and the node to serve as the parent node. Select the cluster containing nodes to be compared with respect to the parent node. You can also start the comparison process by using the drop down node menu.

Shortcut to Node menu: Right click on a node, McDonald's, for instance, for the drop down menu with commands relating to that node to appear as shown in Figure 12.

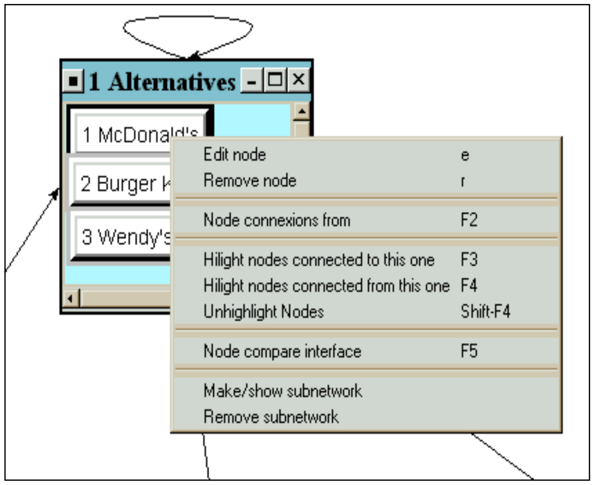


Figure 12. The Dropdown Menu for a Node.

To initiate comparisons with respect to a selected node select the Node Compare Interface command from the drop-down menu, then select the cluster having the nodes you want to compare with respect to McDonald's. This will bring up the comparisons screen in the Questionnaire mode as shown in Figure 13. You can select from several ways to do comparisons: Graphic, Verbal, Matrix, and Questionnaire listed on the tabs at the top.

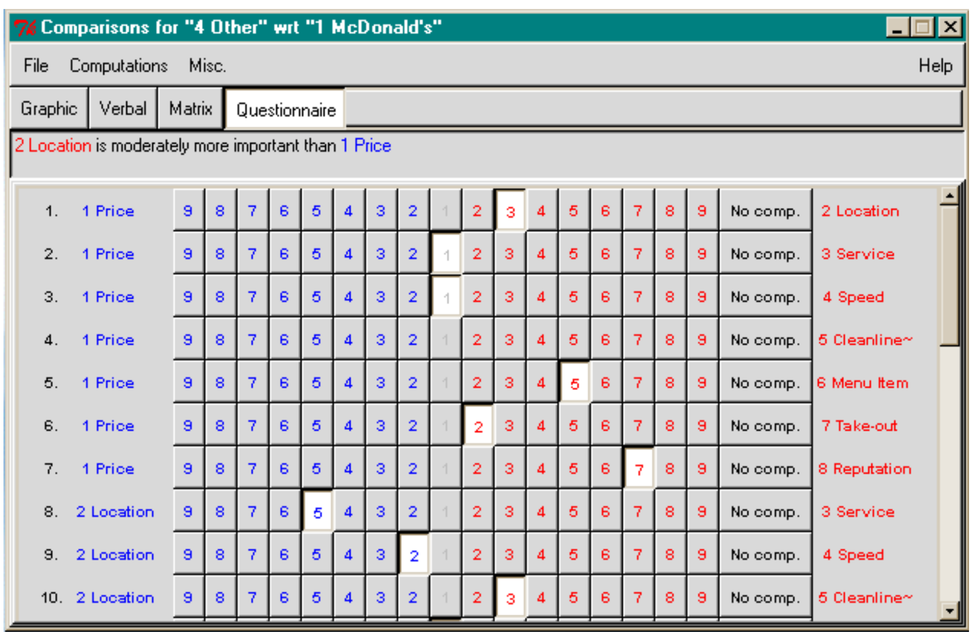


Figure 13. The Questionnaire Mode for Comparing Nodes in the "Other" Cluster with respect to McDonald's

To switch from the Questionnaire mode to the Matrix mode shown in Figure 14 click on the Matrix tab.

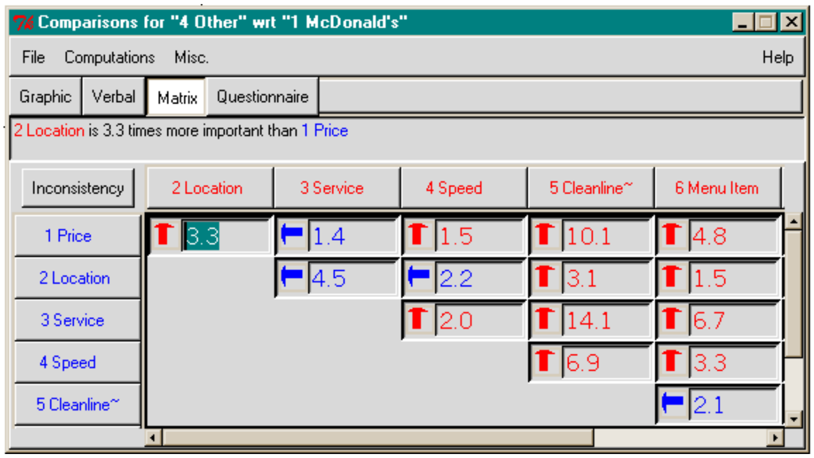


Figure 14. The Matrix Mode for Comparing Nodes in the "Other" Cluster with respect to McDonald's

To show all the judgments in the matrix you must use the scroll bars at the right side and bottom of the window. A judgment is entered in each cell. A cell contains the comparison for the pair listed at the top and at the side. The arrows in the Matrix mode point toward the preferred node of the pair. The top node is preferred when the arrow is red, the side node when the arrow is blue. To toggle a comparison between red and blue, double-click on the arrow button. This inverts the comparison so that the other node is preferred.

Tip: To invert a judgment, double-click on the arrow button associated with it.

To compute the local priorities associated with these judgments, select the Computations, Show New Priorities command. The priorities of the nodes in the Other cluster with respect to McDonald's will be displayed as shown in Figure 15. Consistency can also be improved from the Computations menu. Select the Okay bar at the bottom of the window to return to the Comparison mode. Select File, Save Changes and File, Close to return to the main view.

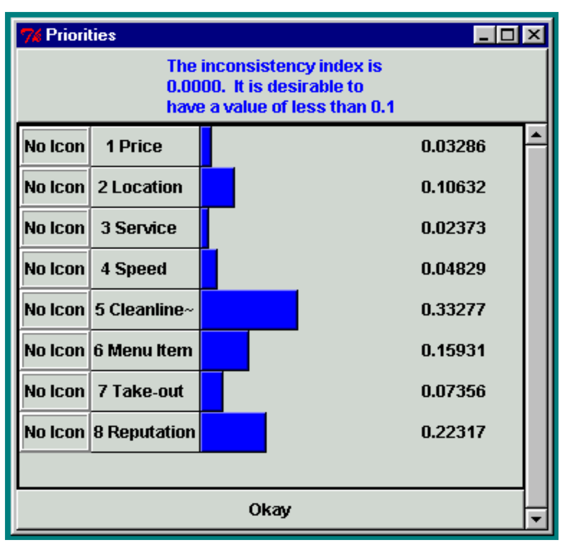


Figure 15. The Local Priorities for Nodes in "Other", Compared with Respect to McDonald's.

The results of all the pairwise comparisons are entered in the unweighted supermatrix. To display it select Computations, Unweighted Supermatrix. To print any of the supermatrices select the command File, Export, and type of supermatrix to be exported, for example, select Unweighted to export to the file *hamburger.unweighted.txt*. This file can then be imported into Excel to format and print as you like by opening Excel and selecting File, Import and enter the name of the text file that was exported. The Excel Text Import Wizard will appear and help you through the process. Select Delimited, then select Tab with the Text Qualifier field set to ", accept the General Data format for columns and finish. The unweighted supermatrix will then be loaded into Excel and will include row and column headings of both clusters and nodes.

Examples are shown below of such imported tables. They were obtained by first exporting from the *SuperDecisions* program, then importing to Excel, formatting within Excel, then importing into Word. The first table gives the cluster weights matrix. The values in the cluster matrix are used to weight the unweighted supermatrix by multiplying the value in the (Alternatives, Alternatives) cell of the cluster matrix times the value in each cell in the (Alternatives, Alternatives) component of the unweighted supermatrix to produce the weighted supermatrix. Every component is weighted with its corresponding Cluster Matrix weight in this way.

Table 1. Cluster Weights Matrix

	Alternatives	Advertising	Quality	Other
1. Alternatives	0.2128	0.2956	0.5000	0.1304
2. Advertising	0.5319	0.2571	0.0000	0.6079
3. Quality	0.0659	0.0000	0.0000	0.0655
4. Other	0.1893	0.4473	0.5000	0.1969

Table 2. Supermatrix of Unweighted Priorities (shown in two parts)

		1 Alternatives			2 Advertising			3Quality of Food		
		1 McDonald's	2 Burger King	3 Wendy's	1 Creativity	2 Promotion	3 Frequency	1 Nutrition	2 Taste	3 Portion
1 Alternati~	1 McDonald's	0.0000	0.8333	0.7500	0.6141	0.7174	0.7174	0.2488	0.2899	0.5989
	2 Burger King	0.8000	0.0000	0.2500	0.2685	0.1942	0.1942	0.1561	0.1040	0.1262
	3 Wendy's	0.2000	0.1667	0.0000	0.1174	0.0884	0.0884	0.5951	0.6061	0.2749
2 Advertisi~	1 Creativity	0.2074	0.1783	0.2810	0.0000	0.3333	0.5000	0.0000	0.0000	0.0000
	2 Promotion	0.1298	0.1120	0.0720	0.1250	0.0000	0.5000	0.0000	0.0000	0.0000
	3 Frequency	0.6628	0.7096	0.6470	0.8750	0.6667	0.0000	0.0000	0.0000	0.0000
3 Quality o~	1 Nutrition	0.3319	0.2810	0.6241	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2 Taste	0.1388	0.0720	0.2823	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3 Portion	0.5293	0.6470	0.0936	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4 Other	1 Price	0.0329	0.2408	0.0300	0.0000	0.8333	0.0000	0.0000	0.0000	0.8571
	2 Location	0.1063	0.2231	0.1417	0.7095	0.0000	0.1958	0.0000	0.0000	0.0000
	3 Service	0.0237	0.1418	0.0648	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4 Speed	0.0483	0.1407	0.0641	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5 Cleanliness	0.3328	0.1096	0.2756	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	6 Menu Item	0.1593	0.0512	0.1571	0.1377	0.1667	0.3108	0.0000	0.0000	0.0000
	7 Take-out	0.0736	0.0506	0.0589	0.0000	0.0000	0.0000	0.0000	0.0000	0.1429
	8 Reputation	0.2232	0.0422	0.2078	0.1528	0.0000	0.4934	0.0000	0.0000	0.0000

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		4 Other							
		1 Price	2 Location	3 Service	4 Speed	5 Cleanliness	6 Menu Item	7 Take-out	8 Reputation
1 Alternati~	1 McDonald's	0.6531	0.6531	0.3319	0.5387	0.2500	0.4934	0.4837	0.6749
	2 Burger Ki~	0.2507	0.2507	0.1388	0.3624	0.2500	0.1958	0.3133	0.2238
	3 Wendy's	0.0962	0.0962	0.5293	0.0989	0.5000	0.3108	0.2029	0.1012
2 Advertisi~	1 Creativity	0.0000	0.0000	0.0000	0.0000	0.0000	0.0780	0.0000	0.0819
	2 Promotion	0.8333	0.0000	0.0000	0.0000	0.0000	0.1711	0.0000	0.3678
	3 Frequency	0.1667	0.0000	0.0000	0.0000	0.0000	0.7509	0.0000	0.5503
3 Quality o~	1 Nutrition	0.1667	0.0000	0.0000	0.0000	0.0000	0.0756	0.0000	0.0936
	2 Taste	0.0000	0.0000	0.0000	0.0000	0.0000	0.6952	0.0000	0.6241
	3 Portion	0.8333	0.0000	0.0000	0.0000	0.0000	0.2292	0.0000	0.2823
4 Other	1 Price	0.0000	0.0000	0.0000	0.0000	0.0000	0.1153	0.0000	0.0627
	2 Location	0.5000	0.0000	0.0981	0.0000	0.1711	0.0526	0.6572	0.2653
	3 Service	0.0000	0.0000	0.0000	0.1873	0.0780	0.0000	0.0548	0.0444
	4 Speed	0.0000	0.0000	0.2857	0.0000	0.7509	0.1946	0.2880	0.0835
	5 Cleanline~	0.0000	0.0000	0.5181	0.0000	0.0000	0.6375	0.0000	0.2378
	6 Menu Item	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1929
	7 Take-out	0.5000	0.0000	0.0000	0.7313	0.0000	0.0000	0.0000	0.0567
	8 Reputation	0.0000	0.0000	0.0981	0.0814	0.0000	0.0000	0.0000	0.0567

Table 3. Weighted Supermatrix

		1 Alternatives			2 Advertising			3 Quality of food		
		1 McDonald's	2 Burger King	3 Wendy's	1 Creativity	2 Promotion	3 Frequency	1 Nutrition	2 Taste	3 Portion
1 Alternati~	1 McDonald's	0.0000	0.1774	0.1596	0.1815	0.2121	0.2121	0.2488	0.2899	0.2995
	2 Burger Ki~	0.1703	0.0000	0.0532	0.0794	0.0574	0.0574	0.1561	0.1040	0.0631
	3 Wendy's	0.0426	0.0355	0.0000	0.0347	0.0261	0.0261	0.5951	0.6061	0.1375
2 Advertisi~	1 Creativity	0.1103	0.0949	0.1495	0.0000	0.0857	0.1286	0.0000	0.0000	0.0000
	2 Promotion	0.0690	0.0596	0.0383	0.0321	0.0000	0.1286	0.0000	0.0000	0.0000
	3 Frequency	0.3526	0.3775	0.3442	0.2250	0.1714	0.0000	0.0000	0.0000	0.0000
3 Quality o~	1 Nutrition	0.0219	0.0185	0.0411	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2 Taste	0.0091	0.0047	0.0186	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3 Portion	0.0349	0.0427	0.0062	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4 Other	1 Price	0.0062	0.0456	0.0057	0.0000	0.3727	0.0000	0.0000	0.0000	0.4286
	2 Location	0.0201	0.0422	0.0268	0.3173	0.0000	0.0876	0.0000	0.0000	0.0000
	3 Service	0.0045	0.0268	0.0123	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4 Speed	0.0091	0.0266	0.0121	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5 Cleanline~	0.0630	0.0207	0.0522	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	6 Menu Item	0.0302	0.0097	0.0297	0.0616	0.0745	0.1390	0.0000	0.0000	0.0000
	7 Take-out	0.0139	0.0096	0.0112	0.0000	0.0000	0.0000	0.0000	0.0000	0.0714
	8 Reputation	0.0422	0.0080	0.0393	0.0684	0.0000	0.2207	0.0000	0.0000	0.0000

		4 Other							
		1 Price	2 Location	3 Service	4 Speed	5 Cleanliness	6 Menu Item	7 Take-out	8 Reputation
1 Alternati~	1 McDonald's	0.0852	0.6531	0.1326	0.2151	0.0998	0.0643	0.1932	0.0880
	2 Burger Ki~	0.0327	0.2507	0.0554	0.1447	0.0998	0.0255	0.1251	0.0292
	3 Wendy's	0.0125	0.0962	0.2114	0.0395	0.1997	0.0405	0.0810	0.0132
2 Advertisi~	1 Creativity	0.0000	0.0000	0.0000	0.0000	0.0000	0.0474	0.0000	0.0498
	2 Promotion	0.5066	0.0000	0.0000	0.0000	0.0000	0.1040	0.0000	0.2236
	3 Frequency	0.1013	0.0000	0.0000	0.0000	0.0000	0.4565	0.0000	0.3345
3 Quality o~	1 Nutrition	0.0109	0.0000	0.0000	0.0000	0.0000	0.0050	0.0000	0.0061
	2 Taste	0.0000	0.0000	0.0000	0.0000	0.0000	0.0456	0.0000	0.0409
	3 Portion	0.0546	0.0000	0.0000	0.0000	0.0000	0.0150	0.0000	0.0185
4 Other	1 Price	0.0000	0.0000	0.0000	0.0000	0.0000	0.0226	0.0000	0.0123
	2 Location	0.0981	0.0000	0.0589	0.0000	0.1028	0.0103	0.3947	0.0520
	3 Service	0.0000	0.0000	0.0000	0.1125	0.0469	0.0000	0.0329	0.0087
	4 Speed	0.0000	0.0000	0.1716	0.0000	0.4510	0.0382	0.1730	0.0164
	5 Cleanline~	0.0000	0.0000	0.3112	0.0000	0.0000	0.1250	0.0000	0.0466
	6 Menu Item	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0378
	7 Take-out	0.0981	0.0000	0.0000	0.4393	0.0000	0.0000	0.0000	0.0111
	8 Reputation	0.0000	0.0000	0.0589	0.0489	0.0000	0.0000	0.0000	0.0111

Table 4. Limiting Supermatrix

		1 Alternatives			2 Advertising			3 Quality of food		
		1 McDonald's	2 Burger King	3 Wendy's	1 Creativity	2 Promotion	3 Frequency	1 Nutrition	2 Taste	3 Portion
1 Alternati~	1 McDonald's	0.1749	0.1749	0.1749	0.1749	0.1749	0.1749	0.1749	0.1749	0.1749
	2 Burger Ki~	0.0883	0.0883	0.0883	0.0883	0.0883	0.0883	0.0883	0.0883	0.0883
	3 Wendy's	0.0520	0.0520	0.0520	0.0520	0.0520	0.0520	0.0520	0.0520	0.0520
2 Advertisi~	1 Creativity	0.0727	0.0727	0.0727	0.0727	0.0727	0.0727	0.0727	0.0727	0.0727
	2 Promotion	0.0878	0.0878	0.0878	0.0878	0.0878	0.0878	0.0878	0.0878	0.0878
	3 Frequency	0.1905	0.1905	0.1905	0.1905	0.1905	0.1905	0.1905	0.1905	0.1905
3 Quality o~	1 Nutrition	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087
	2 Taste	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076
	3 Portion	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145
4 Other	1 Price	0.0462	0.0462	0.0462	0.0462	0.0462	0.0462	0.0462	0.0462	0.0462
	2 Location	0.0681	0.0681	0.0681	0.0681	0.0681	0.0681	0.0681	0.0681	0.0681
	3 Service	0.0091	0.0091	0.0091	0.0091	0.0091	0.0091	0.0091	0.0091	0.0091
	4 Speed	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248
	5 Cleanline~	0.0271	0.0271	0.0271	0.0271	0.0271	0.0271	0.0271	0.0271	0.0271
	6 Menu Item	0.0474	0.0474	0.0474	0.0474	0.0474	0.0474	0.0474	0.0474	0.0474
	7 Take-out	0.0210	0.0210	0.0210	0.0210	0.0210	0.0210	0.0210	0.0210	0.0210
	8 Reputation	0.0596	0.0596	0.0596	0.0596	0.0596	0.0596	0.0596	0.0596	0.0596

		4 Other							
		1 Price	2 Location	3 Service	4 Speed	5 Cleanliness	6 Menu Item	7 Take-out	8 Reputation
1 Alternati~	1 McDonald's	0.1749	0.1749	0.1749	0.1749	0.1749	0.1749	0.1749	0.1749
	2 Burger Ki~	0.0883	0.0883	0.0883	0.0883	0.0883	0.0883	0.0883	0.0883
	3 Wendy's	0.0520	0.0520	0.0520	0.0520	0.0520	0.0520	0.0520	0.0520
2 Advertisi~	1 Creativity	0.0727	0.0727	0.0727	0.0727	0.0727	0.0727	0.0727	0.0727
	2 Promotion	0.0878	0.0878	0.0878	0.0878	0.0878	0.0878	0.0878	0.0878
	3 Frequency	0.1905	0.1905	0.1905	0.1905	0.1905	0.1905	0.1905	0.1905
3 Quality o~	1 Nutrition	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087	0.0087
	2 Taste	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076	0.0076
	3 Portion	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145
4 Other	1 Price	0.0462	0.0462	0.0462	0.0462	0.0462	0.0462	0.0462	0.0462
	2 Location	0.0681	0.0681	0.0681	0.0681	0.0681	0.0681	0.0681	0.0681
	3 Service	0.0091	0.0091	0.0091	0.0091	0.0091	0.0091	0.0091	0.0091
	4 Speed	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248	0.0248
	5 Cleanline~	0.0271	0.0271	0.0271	0.0271	0.0271	0.0271	0.0271	0.0271
	6 Menu Item	0.0474	0.0474	0.0474	0.0474	0.0474	0.0474	0.0474	0.0474
	7 Take-out	0.0210	0.0210	0.0210	0.0210	0.0210	0.0210	0.0210	0.0210
	8 Reputation	0.0596	0.0596	0.0596	0.0596	0.0596	0.0596	0.0596	0.0596

Table 5. Limiting Priorities and Normalized by Cluster Priorities

		Priorities from Limiting Matrix	Priorities Normalized by Cluster
1 Alternatives	1 McDonald's	0.1749	0.5549
	2 Burger King	0.0883	0.2801
	3 Wendy's	0.0520	0.1650
2 Advertising	1 Creativity	0.0727	0.2071
	2 Promotion	0.0878	0.2501
	3 Frequency	0.1905	0.5427
3 Quality of food	1 Nutrition	0.0087	0.2825
	2 Taste	0.0076	0.2468
	3 Portion	0.0145	0.4708
4 Other	1 Price	0.0462	0.1523
	2 Location	0.0681	0.2245
	3 Service	0.0091	0.0300
	4 Speed	0.0248	0.0818
	5 Cleanliness	0.0271	0.0894
	6 Menu Item	0.0474	0.1563
	7 Take-out	0.0210	0.0692
	8 Reputation	0.0596	0.1965

MAKING CLUSTER COMPARISONS

To compare clusters take each cluster in turn (as the parent) and pairwise compare all the clusters it connects to for importance with respect to their influence on it. This is how the Cluster Matrix is generated. Keep in mind that the overall goal here is Market Share. For example, select Assess, Compare, Cluster comparisons and choose the Alternatives cluster. The comparison process now is used to pairwise compare the clusters for influence to which the parent cluster connects. Here is an example of how you formulate a cluster comparison question: "Which influences the market share of the Alternatives more, Advertising or Quality of Food?" The judgment is made that advertising is between very strongly and extremely more important than the quality of food in influencing market share, and a value of 8.1 is entered.

MAKING INNER DEPENDENT CLUSTER COMPARISONS

As the Alternatives cluster is inner dependent, it is connected to itself, so it is one of the four clusters being pairwise compared with respect to Alternatives. The only time you have to ask such a question, comparing a cluster against another cluster with respect to itself as the parent cluster, is when it exhibits inner dependence. Since nodes in the Alternatives cluster are connected to other nodes in that cluster, it must influence itself. If it makes sense to ask the question: "Does Wendy's or Burger King influence McDonald's market share more?" so it needs to be compared for how important its influence is on itself compared to the other clusters it links to. The Wendy's versus Burger King question makes sense. We are really asking which one is a stronger competitor of McDonald's. A judgment of 4 was entered for Burger King over Wendy's; that is, Burger King is a stronger competitor, and that seems reasonable as Burger King has the second largest market share of the three alternatives.

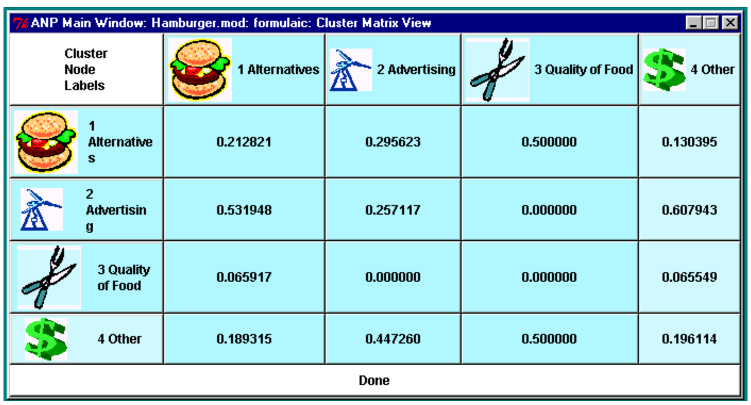
WHY MAKE CLUSTER COMPARISONS?

If all the clusters are equally important it is not necessary to make cluster comparisons, and the cluster weights are set to 1/n in the cluster matrix. The value of n is equal to the number of non-zero components beneath each component across the top of the unweighted supermatrix.

However, the clusters in a network may not be equally important. Then they need to be compared to establish the weights in the cluster matrix. Weighting all the elements in each unweighted supermatrix component by the corresponding cluster matrix cell, whether set by the default value of 1/n described above, or by comparing the clusters and using the derived values, causes the matrix to be column stochastic, that is, each column sums to one. In Table 2 for the unweighted supermatrix the first column sums to 4.000. In Table 3 for the weighted supermatrix it sums to 1.000.

It is essential in real life problems that one know the importance of the groups or clusters to which the elements belong because the final priorities do (and should) depend on that. For example, a society of astronomers is not as important to immediate human survival as the society of farmers, although on the face of it an astronomer may seem more important than a farmer because there are a much smaller number of them (and some people say the greatest potential disaster is a comet crashing into the earth – so astronomers might help avoid that).

Select the Computations, Cluster Matrix command to display the Cluster matrix for the *Hamburger* model shown in Figure 16.


ANP Main Window: Hamburger.mod: formulaic: Cluster Matrix View

Cluster Node Labels	1 Alternatives	2 Advertising	3 Quality of Food	4 Other
1 Alternatives	0.212821	0.295623	0.500000	0.130395
2 Advertising	0.531948	0.257117	0.000000	0.607943
3 Quality of Food	0.065917	0.000000	0.000000	0.065549
4 Other	0.189315	0.447260	0.500000	0.196114

Done

Figure 16. The Cluster Matrix

THE FINAL RESULTS

The final results for the Hamburger model are obtained by selecting the command Computations, Synthesize. The results are shown in Table 5: McDonald's has 55.49% of the market share, Burger King has 28.01% and Wendy's has 16.50%. At the time the model was done the actual values published in the *Market Share Reporter* of 1994 were: 58.23%, 28.57% and 13.20%.

DEMONSTRATION OF A TWO-LAYER SYSTEM, THE CAR PURCHASE BCR MODEL

This model will be used to show a 2-level model with a top-level control network and three sub-networks. It is a model to pick the best type of car: European, Japanese or American. The top-level control network is actually a hierarchy with three control criteria: Benefits, Costs and Risks, and there is a sub-network associated with each. Every sub-network must contain a cluster with the alternatives in it, and these sub-networks do.

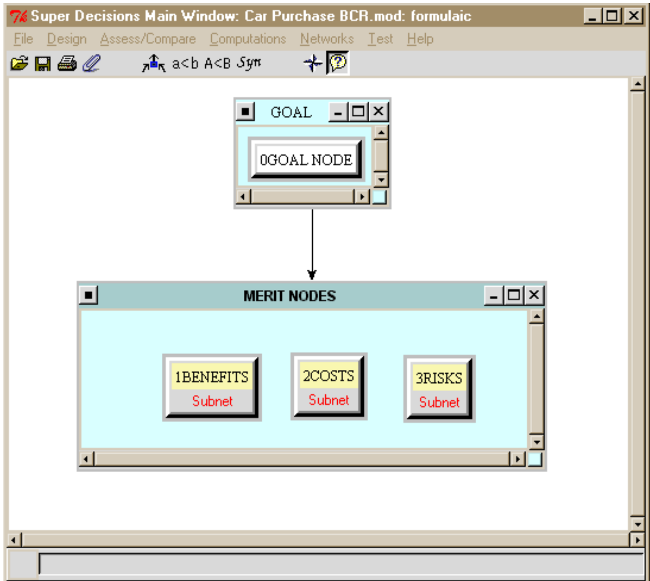


Figure 17. The Top Level Network for the *Car Purchase BCR* Model.

Make/Show Subnets

To make or show a sub-network for a node, right click on the background of the node and select the command Make/show sub-network as shown in the screen clip below. If there is an existing sub-network, it will open. If not, a blank window will appear. When a node has an attached sub-network, the word Subnet is tacked onto the node name. After creating a sub-network, you can open it by left double-clicking on the node name of the node it is attached to. You *must* click on the node name and *not* the red area with the word Subnet on it to bring up the sub-network.

Node Menu

To have a drop-down node menu appear, right-click on any node, for example the Risks node, as shown in Figure 18 below. Commands invoked from this menu will apply to the Risk node.

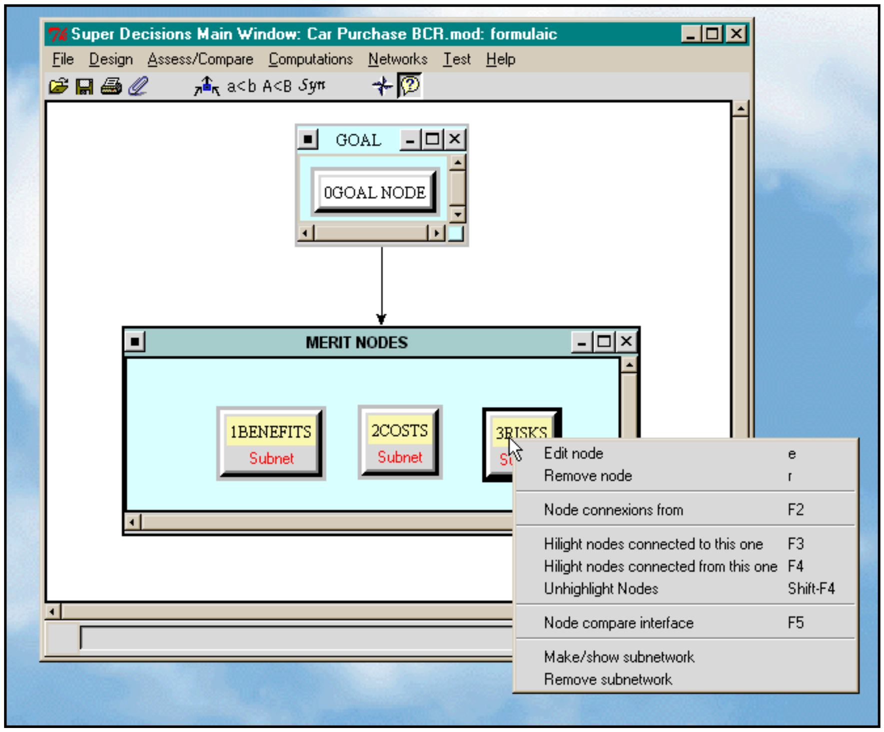


Figure 18. Node Menu Commands

The sub-networks are contained in separate Windows as shown in the view of the entire model in Figure 19 below. To open the subnet attached to the Benefits node, for example, click on the word Benefits (*not* on the red word Subnet beneath it!) Note that the subnets here are decision subnets as each has an Alternatives cluster in it. The command structure in a subnet window is the same as in the top-level network window.

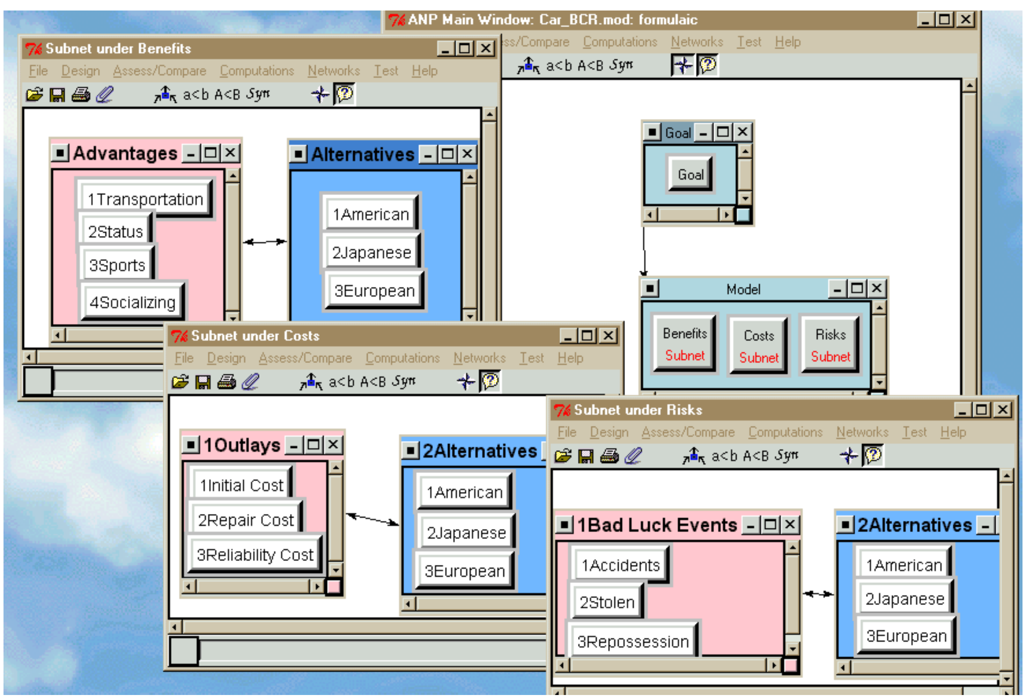


Figure 19. *Car Purchase BCR* Model View of All Networks.

Tip: In handling the subnet windows if one disappears it may have become minimized and be down on the taskbar at the bottom of the desktop screen. Left-click on it to restore it.

Tip on Saving: You can save a complete complex model by selecting the File, Save command from either the top-level network or from a subnet. You can also opt to save only a subnet. You can save subnets and use them as templates later by opening in the blank window for a new subnet.

SYNTHESIZING TO SHOW PRIORITIES IN A SUB-NETWORK

Open the Benefits subnet by double-clicking on the word Benefits. Select the Computations, Synthesize command in the Benefits Subnet to show the results under Benefits. You see that the European car has the greatest benefits as shown in Figure 20.

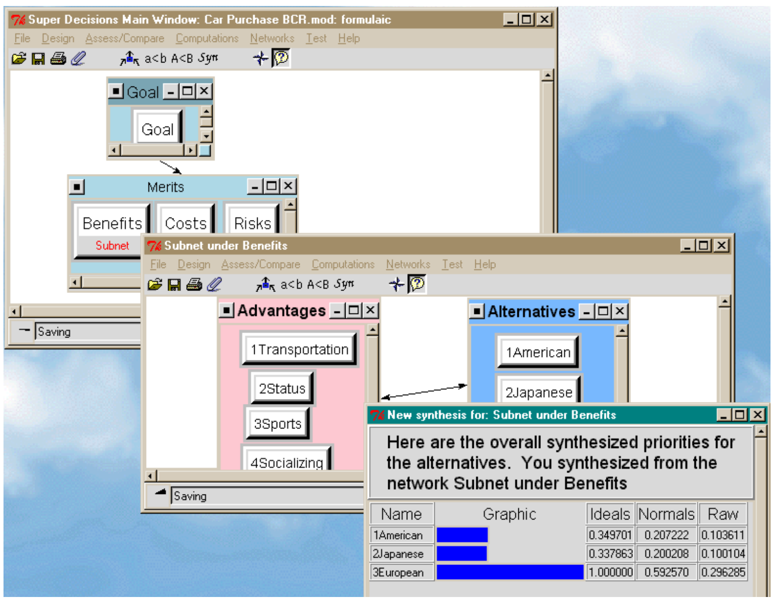
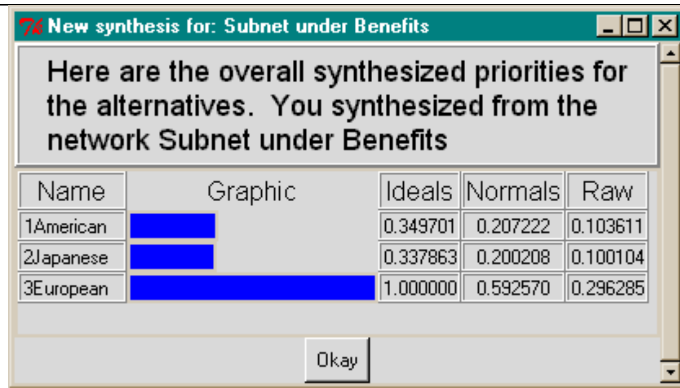
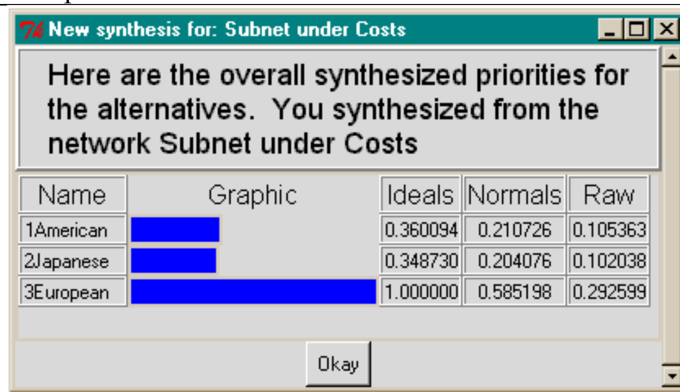


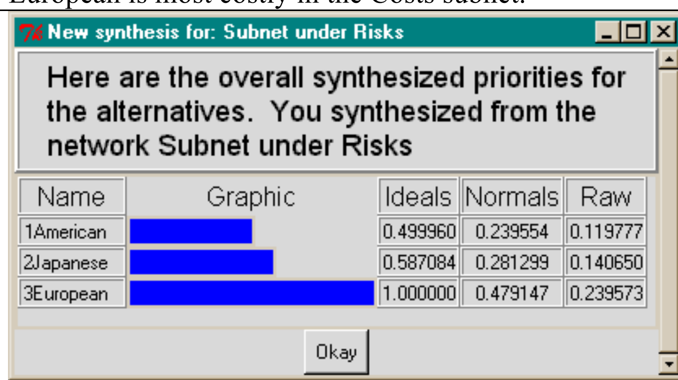
Figure 20. The Synthesis Results in the Benefits Subnet.

The synthesized priorities for each of the three subnets are shown in Table 6 below. In the Costs network and the Risks network the comparison questions are phrased this way: "Which is the most costly?" and "Which is the most risky?" Thus the European car has a 1 in the Ideals column for COSTS in Table 6 which means it is the *most* costly to buy. It also has a 1 in the Ideals column for RISKS so it is also the most risky. The sub-network results are shown in Table 6, and the results as combined in the top-level network in Table 7.

Table 6. Results in the Subnets of the Car Purchase BCR model.

<table>
<tr><td>BENEFITS</td><td>

European is best in the Benefits subnet</td></tr>
<tr><td>COSTS</td><td>

European is most costly in the Costs subnet.</td></tr>
<tr><td>RISKS</td><td>

European is most risky in the Risks subnet.</td></tr>
</table>

The overall results are obtained by synthesizing in the top-level network using the Computations, Synthesize command as shown in Figure 21. This combines the results from the subnets according to the formula (Benefits/(Costs*Risks) for each alternative.

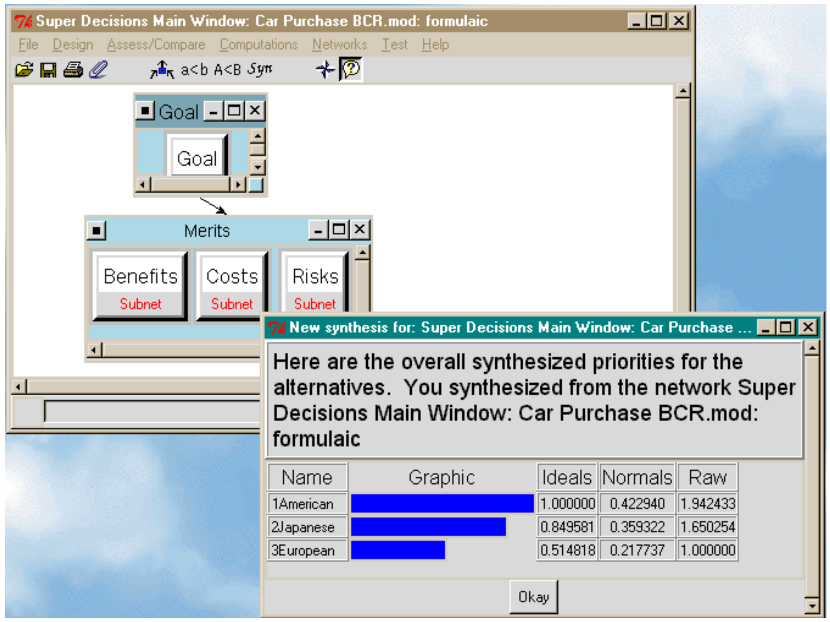


Figure 21. The *Car Purchase BCR* Overall Results.

Select the command Design, Add/Edit Formula shown in Figure 22. This opens the Formula dialogue box shown in Figure 23.

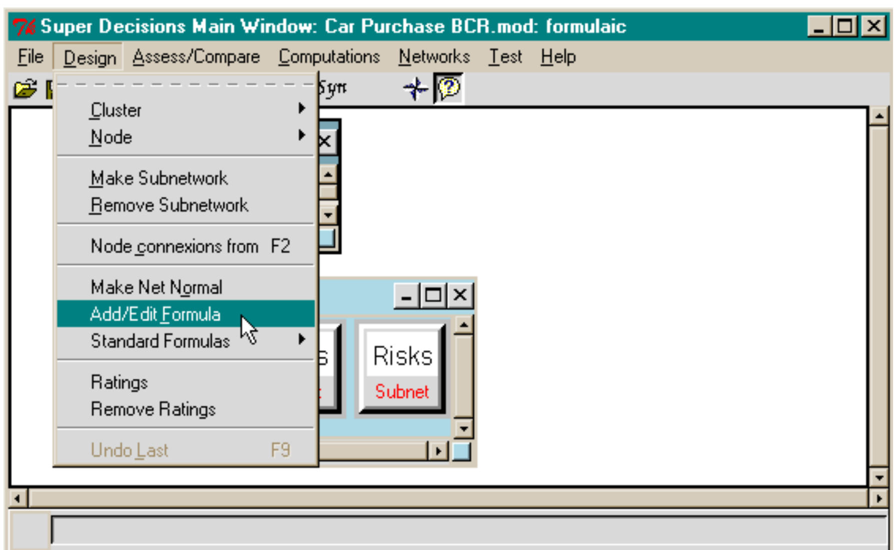


Figure 22. The Menu Command that Shows the Formula.

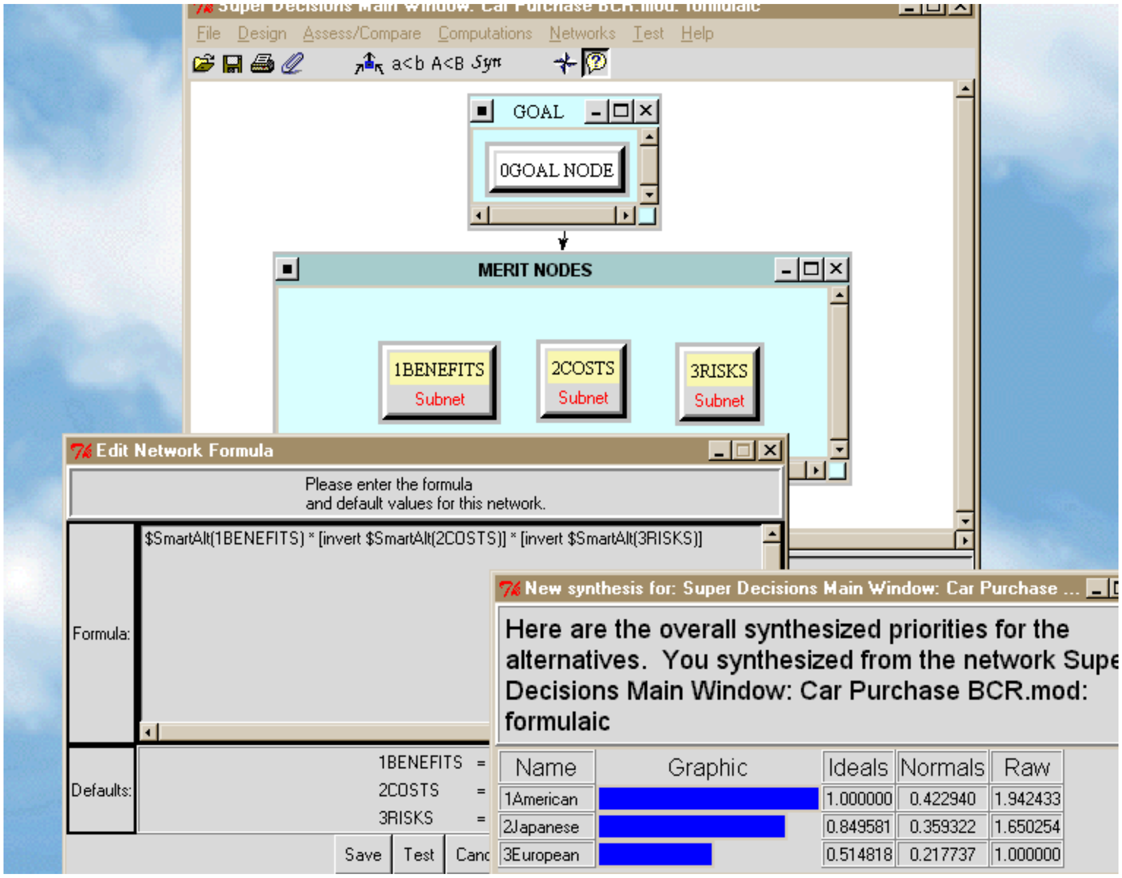


Figure 23. The Multiplicative Formula for Combining Results from Subnets in *Car Purchase BCR* Model

In the term in the formula, $SmartAlt(Benefits), the $ prefix indicates that the term is to be calculated for each alternative and SmartAlt(Benefits) means to use the appropriate values from the network beneath the Benefits node. This gives the results shown in the column under BENEFITS in Table 7.

The Multiplicative Formula is used in Figure 23. The Computations Synthesize command displays results for the alternatives in three ways: as Ideal, Normal and Raw numbers. Smart means use the Raw numbers if the network has more networks beneath it; and use the Ideal numbers if the network is either a bottom level network, or a network that has Ratings. So in this case Smart is choosing the Ideal numbers. The *invert* function in the Costs and Risks terms inverts the values; for example, Costs is inverted to 1/Costs. For more discussion of formulas see the section on Formulas at the end of this chapter.

Now change to the additive formula by selecting the command Design, Standard Formulas, Additive. Synthesis now gives the results shown in Figure 24.

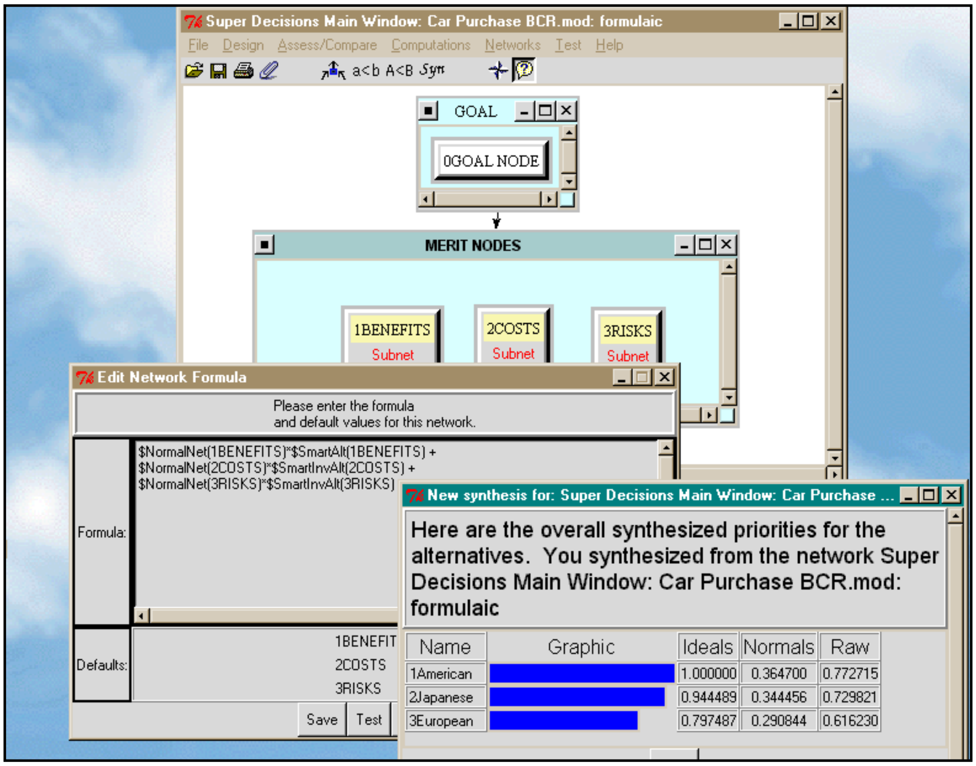


Figure 24. Synthesis Using the Additive Formula

The results are shown in two ways in Table 7: using the multiplicative formula and using the additive formula. In the additive formula the priorities in the Costs and Risks subnets have to be flipped from the most costly (risky) alternative having the highest value to the least costly (risky) alternative having the highest value. See the section on Formulas at the end of the chapter for how to do that.

Table 7. Combining Priorities in the Top-Level Network in Two Ways

	BENEFITS	COSTS	RISKS	B/(C*R) MULTIP. FORMULA	B+1/C+1/R ADDITIVE FORMULA
American Car	.104	.105	.120	. 423	.365
Japanese Car	.100	.102	.141	.359	.344
European Car	.296	.293	.240	.218	.291

Analysis of Results for the Car Purchase BCR Model

To analyze the outcome for the *Car Purchase BCR* model shown in Table 7, we can observe that the European car has the most benefits by far. But it is also the most costly, both to purchase and repair, and the most risky, because of its greater likelihood of being stolen. It has the lowest overall priority. Netting it all out the American car is best over all, mostly because of its lower risks. The ranks are the same for the cars using either formula for synthesis.

Sensitivity Graph

Now we will show how the results would change if the importance of the Benefits node were changed. Make sure the additive formula is selected (the sample model has the formula set to additive). See the sensitivity graph in Figure 25 with Benefits selected as the Independent Parameter. But the sensitivity graph shows that at a Benefits value (on the x-axis) of about .46 the best car changes from American (the top line on the left axis) to Japanese (the second line from the top on the left axis). In the software it is easy to see which line goes with which car when viewing the screen as they are color-coded. Sensitivity can be performed only when the additive formula is selected because in the multiplicative formula the weights of B, C and R are the same for each alternative and cancel out. See the Formulas section at the end of the chapter for more about this.

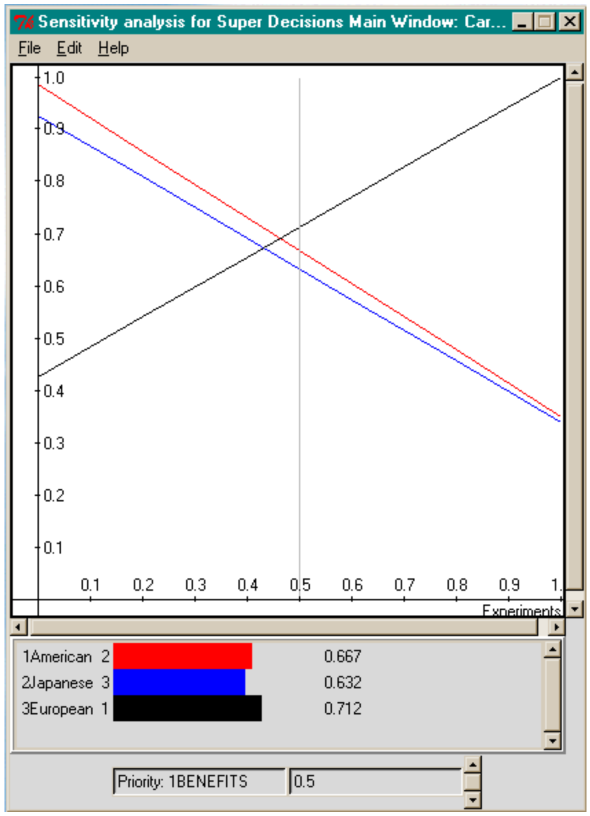


Figure 25. *Car Purchase* Sensitivity Graph for Benefits (Additive Formula)